ABSTRACT
This paper explores the state of the art in FPGA based software radio, and demonstrates the concepts in an example design. The trend in software radios is moving toward an all-digital design within a single chip. Most elements of a digital radio can now be implemented within a DSP or an FPGA. Many vendors provide intellectual property cores, which make up the building blocks of a software radio, but these cores are usually not portable from one vendor to another. This paper presents an example design for a digital radio. An off-the-shelf high-speed analog to digital converter board and a general-purpose FPGA board are used to implement a working radio using portable VHDL based source code and cores.

1. OVERVIEW
Digital radio has been made possible by the combination of two fields. Wireless communications systems were first demonstrated immediately before the twentieth century. Digital processing has more recently matured and begun taking over functions that were traditionally analog in nature.

Implementing parts of a radio receiver in a digital processor has several advantages over an analog solution. A digital receiver may be reconfigured to implement different demodulation and filtering functions on the same hardware platform. Digital filters are not susceptible to component variation, temperature variation, or parasitics. Digitizing a signal also increases its noise immunity. The primary drawbacks of digital receivers are cost and power. As cost and power decrease and speed increases, the digital portion of the radio has been progressively growing from the demodulation stage toward the front end of the receiver.

A radio signal is most uniquely identified by the modulation scheme used to encode data at the carrier frequency for transmission. The most basic type of modulation is amplitude modulation (AM). Frequency modulation (FM) and phase modulation (PM) both fall under the category of angle modulation. Angle modulation has superior performance characteristics to amplitude modulation in several respects. In FM, the baseband signal deviates the frequency of the carrier. For PM, the baseband signal deviates the phase of the carrier. The signal-to-noise ratio is higher for FM and PM signals because interference primarily affects amplitude. FM and PM broadcasting is much more power efficient than AM for the same signal range. The cost of this increased performance is wider bandwidth and higher complexity. This paper focuses on FM and PM.

FM and PM are applicable to transmitting many kinds of information. The most basic baseband signals are analog in nature, such as audio. Digital data signals are also modulated using the same concepts. Digital modulation schemes such as frequency shift keying (FSK) and phase shift keying (PSK) are FM and PM except that the allowable frequencies and phases are set levels to represent discrete symbols.

RF receivers are grouped into two categories: direct conversion (a.k.a. homodyne or zero IF) receivers, which have a single down-conversion stage, and super heterodyne receivers, which have multiple down conversion stages. Two ways to construct a direct conversion receiver are mixing the carrier down to the baseband and demodulating the signal on the carrier directly. Multiple conversion receivers generally select a frequency band and then, in the conversion to an intermediate frequency (IF), select a channel. The signal at the IF is then demodulated to the baseband.
A conventional heterodyne analog radio receiver is implemented with several circuits each with a discrete function. Conventional receiver designs utilize a down conversion stage followed by a demodulator.

Figure 1, General radio receiver block diagram.

The impedance of the antenna is generally different than that of the mixer. An impedance matching network is usually used and may be accompanied by automatic gain control (AGC) or a low-noise amplifier (LNA).

The down conversion stage shifts the frequency of the signal down to the IF by using a mixer. Given carrier and local oscillator signals represented as sinusoids (the $C$ represents that the signals are not coherent),

$$x_C(t) = \sin(\omega_C t + \varphi_C) \quad \text{and} \quad x_{LO}(t) = \sin(\omega_{LO}t).$$

a mixed signal is

$$x_{\text{mix}}(t) = \sin(\omega_C t + \varphi_C) \sin(\omega_{LO}t).$$

Using trigonometric identities, a useful signal is obtained:

$$x_{\text{mix}}(t) = \frac{1}{2} \cos[(\omega_C - \omega_{LO}) t + \varphi_C] - \frac{1}{2} \cos[(\omega_C + \omega_{LO}) t + \varphi_C].$$

The mixed signal is composed of the two frequencies, $\omega_C - \omega_{LO}$ and $\omega_C + \omega_{LO}$. If the local oscillator frequency, $\omega_{LO}$ is chosen to be $\omega_C - \omega_{IF}$, the two resultant frequencies are: $\omega_{IF}$ and $2 \omega_C - \omega_{LO}$.

Image frequencies (those other than $\omega_{IF}$) are filtered out with a bandpass filter. The baseband signal is then recovered by demodulating the IF signal.

If the signal is mixed with a real oscillator, the desired difference frequency components appear as well as undesired sum frequency components. If a complex exponential is used as the local oscillator, the signal spectrum is shifted without the undesired sum frequencies, but both the real and imaginary components are retained until the signal is demodulated. There are several advantages of quadrature mixing including the ability to shift the spectrum down to 0Hz, and the ability to retain phase information through the signal chain.

Mathematically, PM and FM signals may be expressed:

$$x_{\text{PM}}(t) = A \cos[\omega_C t + K_1 s(t) + \varphi_C] \quad \text{and} \quad x_{\text{FM}}(t) = A \cos[(\omega_C + K_2 s(t)) t + \varphi_C],$$

where:

- $A$ is the amplitude of the carrier,
- $\omega_C$ is the frequency of the carrier,
- $s(t)$ is the baseband signal,
- $\varphi_C$ is an arbitrary phase offset,
- $K_1$ is a constant that translates $s(t)$ into radians—the PM index, and
- $K_2$ is a constant translates $s(t)$ into frequency—the deviation constant.

Changing the phase of a signal is analogous to changing its frequency and vise versa. The two modulation schemes are related by differentiation (or integration).

For this paper, we decided to implement a digital FM receiver. This FM receiver could be used to demodulate television audio, broadcast FM, or commercial and government VHF transmission. Many of the concepts discussed are applicable to a wide variety of fields including data communication and even MRI signal processing.

2. FIRST DOWN CONVERSION STAGE

Bandpass Sampling

The introduction of an analog-to-digital converter (ADC) into a receiver allows the possibility for a down-conversion technique impossible in the analog domain: bandpass sampling. Bandpass sampling is a method of sampling below the Nyquist frequency of a signal and aliasing that signal into the sampled signal.
The theoretical requirement for the lowest sampling frequency \( f_s \) to reliably alias the band of interest, \( f_1 < f < f_2 \), into the pass band, \( 0 \leq f < \frac{1}{2} \cdot f_s \), is: \( f_s \geq 2f_2 / \text{floor}(f_2/(f_2 - f_1)) \).

The drawback of bandpass sampling is that noise throughout the spectrum is aliased into the baseband. Even though the resultant sample rate is low, the ADC must have enough bandwidth to pass the signal prior to the desired aliasing in order to avoid aperture effects. All ADC’s have a low-pass filtering nature that results from their sample-and-hold function. An ideal ADC would sample an analog signal instantly, as a delta function. Real ADCs have a sampling aperture \( (T_a) \) that attenuate signals above \( 1/T_a \).

**Digital Control of the Local Oscillator**

Frequency synthesizers or numerically controlled oscillators (NCO) may be used to implement the local oscillator used to mix the carrier down to the IF, in a multiple conversion receiver, or to the baseband, in a direct conversion receiver. The advantage of having a local oscillator controlled by the digital domain is the ease of channel tuning. A programmed parameter sets the selected channel.

**Choosing the IF**

One criteria for choosing an IF frequency is the consideration of rejection of image frequencies. Mixing a frequency band with a local oscillator of \( f_{LO} = f_C - f_{IF} \) will cause two channel frequencies to appear at \( f_{IF} \): the desired channel frequency, \( f_C \), and another frequency \( f_C - 2f_{IF} \). The IF should be selected so that the second candidate \( (f_C - 2f_{IF}) \) does not exist—that is, it falls outside of the band of interest (having bandwidth, BW) and therefore has already been removed by the band selection filter. If \( f_C \) is the largest frequency in the band, \( f_C - 2f_{IF} \) should be below lowest frequency, mathematically:

\[
\begin{align*}
f_C - (f_C - 2f_{IF}) & > BW, \\
f_{IF} & > BW / 2.
\end{align*}
\]

Another criteria for choosing the IF frequency is to maximize sensitivity of the receiver. In order to recover an FM signal, a linear amplitude vs. frequency transfer function (a discriminator) or a linear phase vs. frequency transfer function may be utilized in the demodulation process. If the ratio of the IF frequency to the peak deviation of the signal is very high, a very steep curve must be utilized to convert the frequency deviation of the signal into amplitude or phase respectively. The slope of the curve is relaxed for a lower IF frequency to deviation ratio. In other words it is easier to obtain high sensitivity with a receiver that utilizes a low frequency IF.

**Quadrature Mixing**

As long as images are rejected adequately, down conversion can be accomplished by mixing a real reference oscillator signal with the carrier frequency. More flexibility is achieved by utilizing a complex exponential as the reference oscillator. The complex exponential contains a real cosine component and an imaginary sine component and therefore two multipliers are needed to implement this type of mixing. The multiplication produces quadrature components known as I and Q (in-phase and quadrature). If quadrature mixing is used in the analog domain, two ADC channels are needed in order to acquire the I and Q components. If only one ADC channel is used, the I and Q components may still be obtained in the digital domain by digitally multiplying the acquired signal with a complex exponential. The sine and cosine oscillator components can be made separately, or by phase shifting a single oscillator by \( \pi/2 \). One way to create a \( \pi/2 \) phase shift is via a Hilbert Transformer.

### 3. DIGITAL DETECTION/DEMODULATION METHODS

**Arc-tangent Phase Demodulation**

Arc-tangent phase demodulation may be used in systems when both in-phase and quadrature channels are available. I and Q channels of a PM signal at the IF,

\[
x_{PM}(t) = \sin[\omega_{IF} + \phi(t)].
\]

may be obtained by mixing \( x_{PM}(t) \) with two waveforms that are \( \pi/2 \) out of phase, with the following result:

\[
x_i(t) = \cos[\omega_{IF}] \sin[\omega_{IF} + \phi(t)]
\]

and
\[ x_Q(t) = \sin[\omega_{IF}t] \sin[\omega_{IF} + \varphi(t)]. \]

Using the product-to-sum trigonometric identities yields:

\[ x_I(t) = \frac{1}{2} \sin[\varphi(t)] + \frac{1}{2} \sin[2\omega_{IF} + \varphi(t)] \]
\[ x_Q(t) = \frac{1}{2} \cos[\varphi(t)] - \frac{1}{2} \cos[2\omega_{IF} + \varphi(t)]. \]

If the \(2\omega_{IF}\) components are filtered out and the in-phase component is divided by the quadrature component, a tangent of the modulated signal is obtained:

\[ x_I(t) / x_Q(t) = \frac{1}{2} \sin[\varphi(t)] / \frac{1}{2} \cos[\varphi(t)] = \tan[\varphi(t)]. \]

Taking the arc-tangent of this result, by a look-up table, for example, gives the phase.

The I and Q components may be produced in the analog domain and sampled or computed digitally after sampling \(x_{PM}(t)\).

**Single Bit Continuous Time Demodulator**

A single bit intermediate frequency (IF) can be used if the system is not sampled and is asynchronous. For example, a continuous time single bit demodulator is shown in figure 2.

![Figure 2, Single bit FM demodulator.](image)

The analog circuit applies a \(\pi/2\) phase shift and then varies the phase vs. frequency. The analog quadrature filter circuit is highly sensitive to component variation and normally requires tuning. The xor gate acts as a one-bit multiplier. This is a variation of Bilotti’s quadrature demodulator. The way it works can be understood through trigonometry. If we represent the fundamental of the input to the analog circuit as

\[ \sin(2\pi f_{IF}t) \]

after the \(\pi/2\) phase shift we have

\[ \cos(2\pi f_{IF}t) \]

the bandpass filter creates an additional phase shift which is dependent on frequency, \(\Phi(f)\), yielding

\[ \cos(2\pi f_{IF}t + \Phi(f)) \]

The multiplier creates the product

\[ \sin(2\pi f_{IF}t) \cdot \cos(2\pi f_{IF}t + \Phi(f)) \]

which simplifies to

\[ \frac{1}{2} \sin(-\Phi(f)) + \frac{1}{2} \sin(4\pi f_{IF}t + \Phi(f)) \]

The second term can be removed with an analog lowpass filter or integrator. For small \(\Phi(f)\) variation, the first term can be approximated by \(-\frac{1}{2} \cdot \Phi(f)\), which is a signal that varies in amplitude with changes in frequency, i.e., the recovered baseband signal.

The single bit architecture can also operate synchronously and sampled. The sample rate must be very high or phase locked to the IF carrier.

**Zero Crossing Demodulation**

Counting the number of zero crossings in a signal per unit time gives an indication of instantaneous frequency. Limiting the IF signal gives a single-bit indication of the modulated signal. The single bit IF signal can be input into the clock input of a counter. The counter determines the number of zero crossing events per unit sample time. The nominal IF count is subtracted from the count and the result is scaled to form the demodulated output.

The resolution obtained by this method is the ratio of the signal bandwidth to the sample frequency of the baseband, independent of the IF. This resolution is too low for narrow band FM demodulation.

A more useful method of using a single bit IF signal is to measure its period in sample counts. Taking the inverse of the period gives the instantaneous frequency. The minimum sampling
frequency is determined by the required resolution of the demodulated signal.

If \( m \) is the resolution measured at each \( T_{IF} \) (the period of the IF) and \( BW_{MOD} \) is the bandwidth of the modulated signal, the minimum sample frequency may be approximated:

\[
f_s = m \cdot \left( \frac{f_{IF}}{BW_{MOD}} \right) \cdot f_{IF},
\]

The number of bits of resolution (\( n \)) at the desired sample rate of the baseband (\( f_{BB} \)) depends on \( m \) and the ratio of \( T_{BB} \) (the period of \( f_{BB} \)) to \( T_{IF} \) or \( f_{IF} / f_{BB} \):

\[
n = \log_2 \left[ m \left( \frac{T_{BB}}{T_{IF}} \right) \right].
\]

Using the later equation in the expression of \( f_s \), the minimum sample frequency for a resolution of \( n \) bits at the baseband sample frequency becomes:

\[
f_s = 2^n \cdot f_{IF}^2 / \left( BW_{MOD} \cdot \left( \frac{T_{BB}}{T_{IF}} \right) \right),
\]

which simplifies to:

\[
f_s = 2^n \cdot f_{IF} \cdot f_{BB} / BW_{MOD}.
\]

If this method is used to get 8-bit resolution of an FM audio signal (\( BW_{MOD} = 75\text{kHz} \), \( f_{BB} = 40\text{kHz} \)) from a IF of 455kHz, the single bit IF must be sampled above 62MHz.

Time Verses Amplitude Resolution

Amplitude resolution is related to time resolution in sampled systems. For an FM system, where a limiter is used to remove amplitude information, time resolution must be preserved. The demodulation process needs to achieve a resolution often defined in bits. For a single bit sampled system, this resolution can be achieved by using a sampling rate, \( f_s \sim 2^{N1} / TD \) where TD is the deviation change of the period and \( N1 \) is the number of bits of resolution required in the demodulated signal. For an ADC based system, the sample rate of a sinusoidal IF can be reduced by the resolution of the ADC, provided it meets the Nyquist sampling criteria of at least twice the IF channel bandwidth. Either bandpass under sampling or lowpass antialiasing techniques can be used. The sampling rate can be expressed as

\[
f_s \sim 2^{(N1-N2)} / TD, \text{ where } N2 \text{ is the number of ADC bits}.
\]

Discrete-time Quadrature Detection

A block diagram of a quadrature detector is shown in figure 3.

![Figure 3, Quadrature demodulator](image)

This detector can be implemented in discrete time by using a Hilbert transformer for the phase shift block and an IIR bandpass filter to perform the frequency to phase function. If the IF signal is already in quadrature form, the \( \pi/2 \) phase shift is not necessary.

Combining a digital \( \pi/2 \) phase shift network and digital biquad bandpass filter, we can implement a modified, discrete-time version of Bilotti’s demodulator.

![Figure 4, Digital implementation of Bilotti’s demodulator](image)

A lowpass filter or integrator stage is used after the demodulator to smooth the recovered baseband signal and eliminate the high frequency products.

4. DEMONSTRATION SYSTEM

A demonstration system was built in order to evaluate the down conversion and demodulation methods described in this paper. A block diagram of the demonstration system is shown in figure 5.
The first block matches the antenna to the input impedance of the mixer, and also selects the band containing the channels of interest. The local oscillator operates at a frequency that is set to the difference between the IF frequency and the selected channel frequency. The local oscillator may be numerically controlled (NCO) by the FPGA or may operate at a fixed frequency selected by a crystal. The IF output is passed to the FPGA in both single bit form and 14 bit form to allow for investigation of various demodulation techniques. The 14 bit signal is from a high speed ADC which can be sampled normally or under sampled to experiment with bandpass sampling down conversion. After the FPGA performs the down conversion, the output is sent to a DAC and amplified to drive a speaker.

Information on the boards used for the demonstration system, the schematic diagram of the front-end board, the VHDL code and results are posted at www.cepdinc.com/gspx04.

5. CONCLUSION
This paper presented an overview of down conversion and demodulation techniques. Many of these techniques lend themselves to implementation in digital form or discrete time systems. An FPGA based demonstration system was constructed using off-the-shelf evaluation boards in order to evaluate several of the methods described in the paper.

6. REFERENCES